Section 5.3
Definite Integrals and Antiderivatives

Rules For Definite Integrals

1. *Order of Integration:* $\displaystyle\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

2. *Zero:* $\displaystyle\int_a^a f(x)\,dx = 0$

3. *Constant Multiple:* $\displaystyle\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$

$$\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx \qquad k = -1$$

4. *Sum and Difference:* $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. *Additivity:* $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

6. *Max-Min Inequality:* If max f and min f are the maximum and minimum values of f on $[a, b]$, then

$$\min f \cdot (b - a) \le \int_a^b f(x)\,dx \le \max f \cdot (b - a).$$

7. *Domination:* $f(x) \ge g(x)$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

$$f(x) \ge 0 \text{ on } [a, b] \Rightarrow \int_a^b f(x)\,dx \ge 0$$

Using the Rules for Definite Integrals

Suppose

$$\int_{-1}^{1} f(x)\,dx = 5, \quad \int_{1}^{4} f(x)\,dx = -2, \quad \text{and} \quad \int_{-1}^{1} h(x)\,dx = 7.$$

Find each of the following integrals, if possible.

(a) $\displaystyle\int_{4}^{1} f(x)\,dx$ **(b)** $\displaystyle\int_{-1}^{4} f(x)\,dx$ **(c)** $\displaystyle\int_{-1}^{1} [2f(x) + 3h(x)]\,dx$

(d) $\displaystyle\int_{0}^{1} f(x)\,dx$ **(e)** $\displaystyle\int_{-2}^{2} h(x)\,dx$ **(f)** $\displaystyle\int_{-1}^{4} [f(x) + h(x)]\,dx$

Show that the value of $\int_0^1 \sqrt{1 + \cos x}\, dx$ is less than 3/2.

Show that $\int_{0}^{\pi} \sin x \, dx$ cannot equal 4

Average (Mean) Value

If f is integrable on [a,b], its average (mean) value on [a,b] is

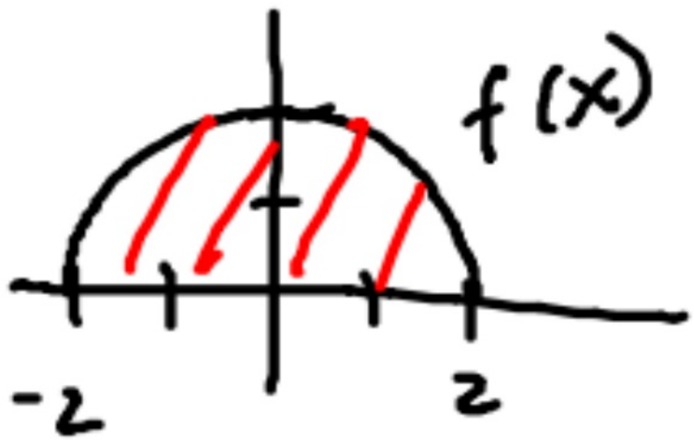
$$av(f) = \frac{1}{b - a} \int_a^b f(x)\, dx$$

Average y value.......

NOT average rate of change!!!

Find the average value of $f(x) = 4 - x^2$ on [0,3]. Does f actually take on this value at some point in the given interval?

Find the average value of f(x) on [-2,2]



Find the average value of $f(x) = 3x^3 - x^2 + 1$ on $[-1,7]$

Mean Value Theorem for Integrals

If f is continuous on [a,b] then there is a c in [a,b] such that

$$f(c) = \frac{1}{b - a} \int_a^b f(x)\,dx$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

The integral is an antiderivative of f

If F is any antiderivative of f, then

$$\int_a^x f(t)\,dt = F(x) + C$$

for some constant C

Substituting a in for x gives the following

$$\int_a^a f(t)\,dt = F(a) + C$$

$$0 = F(a) + C$$

$$C = -F(a)$$

Therefore

$$\int_a^x f(t)\,dt = F(x) - F(a)$$

Find $\int_0^{\pi} \sin x \, dx$ using the formula

$$\int_a^x f(t)\, dt = F(x) - F(a)$$

$F(x) = -\cos x$

$= F(\pi) - F(0)$

$= -\cos(\pi) - (-\cos 0)$

$= 1 + 1 = 2$

pg. 241

19 $\displaystyle\int_{\pi}^{2\pi} \sin x \, dx$

$F(x) = -\cos x$

$= F(2\pi) - F(\pi)$

$= -\cos(2\pi) - (-\cos \pi)$

$= -2$

20

$$\int_0^{\frac{\pi}{2}} \cos x \, dx$$

$$F(x) = \sin x$$

$$= F\left(\frac{\pi}{2}\right) - F(0)$$

$$= \cos\left(\frac{\pi}{2}\right) - \cos(0)$$

$$= 1$$

21

$$\int_0^1 e^x\,dx$$

$F(x) = e^x$

$= F(1) - F(0)$

$= e^1 - e^0$

$= e - 1$

$$\int_0^{\frac{\pi}{4}} \sec^2 x\,dx =$$

$$F(x) = \tan x$$

$$= \tan\left(\frac{\pi}{4}\right) - \tan(0)$$

$$= 1$$

23 $\int_1^4 2x\,dx$

$F(x) = x^2$

$= F(4) - F(1)$

$= 16 - 1$

$= 15$

24 $\int_{-1}^{2} 3x^2\,dx$

$F(x) = \frac{3x^3}{3} = x^3$

$= F(2) - F(-1)$

$= 2^3 - (-1)^3$

$= 9$

27 $\displaystyle\int_{-1}^{1} \frac{1}{1+x^2}\,dx$

$F(x) = \tan^{-1}(x)$

$= \tan^{-1}(1) - \tan^{-1}(-1)$

$=$

28

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$$

29 $\displaystyle\int_1^e \frac{1}{x}\,dx$

$= \ln x$

$= \ln e - \ln 1$

$= 1 - 0$

$= 1$

30

$$\int_1^4 -x^{-2}\,dx$$

$$f(x) = x^{-1} = \frac{1}{x}$$

$$= \frac{1}{4} - \frac{1}{1}$$

$$= -\frac{3}{4}$$